HOSSAM GHANEM

(7) 4.4 *Limits involving infinity (A)*

(1)Main Rules

$\lim\limits_{x\to\infty}\frac{1}{x}=0$	$\lim\limits_{x\to\infty}x=\infty$
$\lim\limits_{x\to\infty}\frac{1}{x^r}=0\ ,\ r>0$	$\lim\limits_{x\to\infty}x^r=\infty\ ,\ r>0$

$\lim\limits_{x\to\infty}\frac{1}{r^x}=0$	$r>1$	$\lim\limits_{x\to\infty}r^x=\infty$
$\lim\limits_{x\to\infty}\frac{1}{5^x}=0$	Ex.	$\lim\limits_{x\to\infty}7^x=\infty$

$\lim\limits_{x\to\infty}\left(\frac{a}{b}\right)^x=0$	$b>a$ $a>0$	$\lim\limits_{x\to\infty}\left(\frac{b}{a}\right)^x=\infty$
$\lim\limits_{x\to\infty}\left(\frac{2}{3}\right)^x=0$	Ex.	$\lim\limits_{x\to\infty}\left(\frac{7}{5}\right)^x=\infty$

(2) quotient

Numerator degree = Denominator degree درجة البسط تساوي درجة المقام	$\lim\limits_{x\to\infty}\frac{4x^3-5x+1}{7x^3+2x-3}=\frac{4}{7}$
Numerator degree > Denominator degree درجة البسط أكبر من درجة المقام	$\lim\limits_{x\to\infty}\frac{2x^5-3x+11}{27x^2+9x-5}=\infty$ ∞
Numerator degree < Denominator degree درجة البسط أصغر من درجة المقام	$\lim\limits_{x\to\infty}\frac{3x^4+7x+11}{7x^5-x-4}=0$ 0

HOSSAM GHANEM

LIMITS INVOLVING $\pm\infty$

$\lim_{x\to\pm\infty} f(x)$

تحتوي على $\sqrt{x^2}$

نأخذ x^2 من تحت الجذر عامل مشترك فتخرج خارج الجذر $|x|$

تحتوي على $|x|$

as $x \to \infty$, $|x| = x$

as $x \to -\infty$, $|x| = -x$

تحتوي على $\sqrt[3]{x}$

نقسم على أكبر أس مع العلم بأن

$x = \sqrt[3]{x^3}$

$\infty - \infty$

بالضرب في المرافق ثم أخذ العامل المشترك إن لزم الأمر مع العلم بأن

$\sqrt{x^2} = |x|$

$\frac{\text{polynomial}}{\text{polynomial}}$ $\frac{\text{حدودية}}{\text{حدودية}}$

$\frac{1}{x^r} = 0$ قسم على أكبر أس ثم يكون كل

$$\lim_{x\to\infty} \frac{4x^3 - 5x + 1}{7x^3 + 2x - 3} = \lim_{x\to\infty} \frac{4 - \frac{5}{x^2} + \frac{1}{x^3}}{7 + \frac{2}{x^2} - \frac{3}{x^3}} = \frac{4}{7}$$

$$\lim_{x\to\infty} \frac{2x^5 - 3x + 11}{27x^2 + 9x - 5} = \lim_{x\to\infty} \frac{2 - \frac{3}{x^4} + \frac{11}{x^5}}{\frac{27}{x^3} + \frac{9}{x^4} - \frac{5}{x^5}} = \frac{2}{0} = \infty$$

$$\lim_{x\to\infty} \frac{3x^4 + 7x + 11}{7x^5 - x - 4} = \lim_{x\to\infty} \frac{\frac{3}{x} + \frac{7}{x^4} + \frac{11}{x^5}}{7 - \frac{1}{x^4} - \frac{4}{x^5}} = \frac{0}{7} = 0$$

$\lim_{x\to a} f(x) = \pm\infty$

$\lim_{x\to 5^-} \frac{1}{x-5} = -\infty$

عند التعويض بالعدد 5 في الدالة يكون الناتج $\frac{1}{0}$ أي ∞ ولمعرفة الإشارة نعوض بعدد أصغر قليلا من 5 وليكن 4 فتكون الإشارة سالبة وبذلك يكون الناتج $(-\infty)$

$\lim_{x\to 5^+} \frac{1}{x-5} = \infty$

عند التعويض بالعدد 5 في الدالة يكون الناتج $\frac{1}{0}$ أي ∞ ولمعرفة الإشارة نعوض بعدد أكبر قليلا من 5 وليكن 6 فتكون الإشارة موجبة وبذلك يكون الناتج (∞)

Example 1	Evaluate the following limit $\lim_{x\to\infty}\frac{\sqrt{x}}{x}$

Solution

$$L=\lim_{x\to\infty}\frac{\sqrt{x}}{x}=\frac{\infty}{\infty}$$

$$L=\lim_{x\to\infty}\frac{\sqrt{x}}{x}=\lim_{x\to\infty}\frac{\sqrt{x}}{\sqrt{x}\sqrt{x}}=\lim_{x\to\infty}\frac{1}{\sqrt{x}}=0$$

Example 2	Evaluate the following limit $\lim_{x\to\infty}\frac{x^3+4x}{8x-2x^3}$

Solution

$$L=\lim_{x\to\infty}\frac{x^3+4x}{8x-2x^3}=\frac{\infty}{\infty}$$

$$L=\lim_{x\to\infty}\frac{x^3+4x}{8x-2x^3}=\lim_{x\to\infty}\frac{1+\frac{4}{x^2}}{\frac{8}{x^2}-2}=\frac{1}{-2}=-\frac{1}{2}$$

Example 3	Evaluate the following limit $\lim_{x\to\infty}\frac{\sqrt[3]{x+8}-2}{x}$

Solution

$$L=\lim_{x\to\infty}\frac{\sqrt[3]{x+8}-2}{x}=\frac{\infty}{\infty}$$

$$L=\lim_{x\to\infty}\frac{\sqrt[3]{x+8}-2}{x}=\lim_{x\to\infty}\frac{\sqrt[3]{\frac{1}{x^2}+\frac{8}{x^3}}-\frac{2}{x}}{1}=\frac{0}{1}=0$$

Example 4 21 May 27. 2001	Evaluate the following limit $\lim_{x\to\infty}\sqrt[3]{\frac{1-8x^3}{x(x^2+1)}}$

Solution

$$L=\lim_{x\to\infty}\sqrt[3]{\frac{1-8x^3}{x(x^2+1)}}=\frac{-\infty}{\infty}$$

$$L=\lim_{x\to\infty}\sqrt[3]{\frac{1-8x^3}{x(x^2+1)}}=\lim_{x\to\infty}\sqrt[3]{\frac{1-8x^3}{x^3+x}}=\lim_{x\to\infty}\sqrt[3]{\frac{\frac{1}{x^3}-8}{1+\frac{1}{x^2}}}=\sqrt[3]{\frac{-8}{1}}=-2$$

Example 5

36 January 17, 2010

Evaluate the following limit $\lim\limits_{x\to-\infty}\dfrac{\sqrt{1+x^2}}{2x+1}$

Solution

$$L=\lim_{x\to-\infty}\frac{\sqrt{1+x^2}}{2x+1}=\frac{\infty}{\infty}$$

$$L=\lim_{x\to-\infty}\frac{\sqrt{1+x^2}}{2x+1}=\lim_{x\to-\infty}\frac{|x|\sqrt{\frac{1}{x^2}+1}}{2x+1}=\lim_{x\to-\infty}\frac{-x\sqrt{\frac{1}{x^2}+1}}{2x+1}=\lim_{x\to-\infty}\frac{-\sqrt{\frac{1}{x^2}+1}}{2+\frac{1}{x}}=\frac{-1}{2}$$

Example 6

Evaluate the following limit $\lim\limits_{x\to\infty}\dfrac{5^x+4}{8-3^x}$

Solution

$$L=\lim_{x\to\infty}\frac{5^x+4}{8-3^x}=\lim_{x\to\infty}\frac{1+\frac{4}{5^x}}{\frac{8}{5^x}-\left(\frac{3}{5}\right)^x}=\frac{1}{0}=\infty$$

Example 7

44 November 9, 2006

Evaluate the following limit $\lim\limits_{x\to\infty}\sqrt{2x^2+3}-\sqrt{2x^2-5}$

Solution

$$L=\lim_{x\to\infty}\left(\sqrt{2x^2+3}-\sqrt{2x^2-5}\right)=\infty-\infty$$

$$L=\lim_{x\to\infty}\frac{\left(\sqrt{2x^2+3}-\sqrt{2x^2-5}\right)\left(\sqrt{2x^2+3}+\sqrt{2x^2-5}\right)}{\sqrt{2x^2+3}+\sqrt{2x^2-5}}=\lim_{x\to\infty}\frac{2x^2+3-(2x^2-5)}{\sqrt{2x^2+3}+\sqrt{2x^2-5}}$$

$$=\lim_{x\to\infty}\frac{2x^2+3-2x^2+5}{\sqrt{2x^2+3}+\sqrt{2x^2-5}}=\lim_{x\to\infty}\frac{8}{\sqrt{2x^2+3}+\sqrt{2x^2-5}}=\frac{0}{\infty}=0$$

Example 8

9 January 8, 1994

Evaluate the following limit $\lim\limits_{x\to\infty}\left(\sqrt{x^2+2x}-x\right)$

Solution

$$L=\lim_{x\to\infty}\left(\sqrt{x^2+2x}-x\right)=\infty-\infty$$

$$L=\lim_{x\to\infty}\frac{\left(\sqrt{x^2+2x}-x\right)\left(\sqrt{x^2+2x}+x\right)}{\sqrt{x^2+2x}+x}=\lim_{x\to\infty}\frac{(x^2+2x)-x^2}{\sqrt{x^2+2x}+x}=\lim_{x\to\infty}\frac{2x}{\sqrt{x^2+2x}+x}$$

$$=\lim_{x\to\infty}\frac{2x}{|x|\sqrt{1+\frac{2}{x}}+x}=\lim_{x\to\infty}\frac{2x}{x\sqrt{1+\frac{2}{x}}+x}=\lim_{x\to\infty}\frac{2}{\sqrt{1+\frac{2}{x}}+1}=\frac{2}{1+1}=1$$

Homework

1	Evaluate the following limit	$\lim_{x\to\infty} x-\sqrt{x^2-3x}$	4 May 19, 1992
2	Evaluate the following limit	$\lim_{x\to\infty}\left(\sqrt{x+9}-\sqrt{x}\right)$	10 June 6, 1994
3	(2pts) Evaluate the following limits, if they exist.	$\lim_{x\to-\infty} x+\sqrt{x^2+3x+1}$	55 April 8, 2010
4	Evaluate the following limits, if they exist:	$\lim_{x\to\infty}\left(\sqrt{x^2+x}-x\right)$ (2 pts.)	57 November 8, 2010
5	[2 pts.] Find	$\lim_{x\to\infty}\left(2x-\sqrt{4x^2+5x}\right)$	39 5 June, 2011
6	Evaluate the following limit	$\lim_{x\to\infty}\frac{10-3^x}{20-3^x}$	
7	Evaluate the following limit	$\lim_{x\to\infty}\frac{2^x+4}{8-7^x}$	
9	Evaluate the following limit	$\lim_{x\to\infty} x-\sqrt{x^2+1}$	

6	Evaluate the following limit $\lim_{x\to\infty}\frac{10-3^x}{20-3^x}$

Solution

$$L=\lim_{x\to\infty}\frac{10-3^x}{20-3^x}=\lim_{x\to\infty}\frac{\frac{10}{3^x}-1}{\frac{20}{3^x}-1}=\frac{-1}{-1}=1$$

7	Evaluate the following limit $\lim_{x\to\infty}\frac{2^x+4}{8-7^x}$

Solution

$$L=\lim_{x\to\infty}\frac{2^x+4}{8-7^x}=\lim_{x\to\infty}\frac{\frac{2^x}{7^x}+\frac{4}{7^x}}{\frac{8}{7^x}-1}=\lim_{x\to\infty}\frac{\left(\frac{2}{7}\right)^x+\frac{4}{7^x}}{\frac{8}{7^x}-1}=\frac{0+0}{0-1}=\frac{0}{-1}=0$$